The maximum number of linearly Independent vectors in V is called dimension of V.

A linearly independent set in V consisting of maximum possible number of vectors in V is basis of V.

Basis: Any sub set S of a vector space V is called basis of V, if hold the following properties.

1. S is linearly independent
2. S generates V i.e $V = L(S)$

the span $L(S)$ is linear Combination of vectors in S.

$$\text{Span}(S) \;/\; L(S) = \{c_1 v_1 + c_2 v_2 \cdots c_n v_n\}$$

where $c_1\, c_2 \cdots c_n$ are scalar.

let $v_1 = (1,2,1)$, $v_2 = (2,9,0)$ and $v_3 = (3,3,4)$

Show that $S = \{v_1, v_2, v_3\}$ is basis of R^3

$b = (b_1 b_2 b_3)$ $\exists$ $c_1 c_2 c_3 \in R$

$(b_1, b_2, b_3) = c_1 v_1 + c_2 v_2 + c_3 v_3$

$(b_1, b_2, b_3) = c_1(1,2,1) + c_2(2,9,0) + c_3(3,3,4)$

$c_1 + 2c_2 + 3c_3 = b_1$

$2c_1 + 9c_2 + 3c_3 = b_2$

$c_1 + 4c_3 = b_3$

To prove that vectors are L.I

$c_1 + 2c_2 + 3c_3 = 0$

$2c_1 + 9c_2 + 3c_3 = 0$

$c_1 + 4c_3 = 0$

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

$|A| = 1(36-0) - 2(8-3) + 3(0-9)$

$= 36 - 10 - 27 = -1 \neq 0$. L.I

$|A| = -1 \neq 0$

The vectors are linearly independent and hence vectors form a basis for R^3

Show that $S = \{x^2+1, x-1, 2x+2\}$ is a basis of $\mathbb{R}^3$

let $\alpha_1 \alpha_2 \alpha_3 \in R$

$$\alpha_1(x^2+1) + \alpha_2(x-1) + \alpha_3(2x+2) = \underline{0,0,0}$$

$\alpha_1 x^2 = 0$

$\alpha_1 = 0$ — (1)

$\alpha_1 - \alpha_2 + 2\alpha_3 = 0$ — (2)

$\alpha_2 + 2\alpha_3 = 0$ — (3)

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$$

from (2) $\alpha_1 = 0$

$-\alpha_2 + 2\alpha_3 = 0$

$2\alpha_3 = \alpha_2$

from eq (3) $\alpha_2 + 2\alpha_3 = 0$

$2\alpha_3 + 2\alpha_3 = 0$

$4\alpha_3 = 0$

$\alpha_3 = 0 \implies \alpha_2 = 0$

$\Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$

Vectors are linearly independent

Show that the vectors $(1,0,0)$, $(1,1,0)$ and $(1,1,1)$ form a basis for $\underline{R}^3$

let $S = \{(1,0,0), (1,1,0), (1,1,1)\}$

let $a_1\ a_2\ a_3 \in R$

$a_1(1,0,0) + a_2(1,1,0) + a_3(1,1,1) = 0$

$a_1 + a_2 + a_3 = 0$

$a_2 + a_3 = 0$

$a_3 = 0$

$a_3 = 0 \Rightarrow a_2 = 0$

$a_2 = 0 \Rightarrow a_1 = 0$

$a_1 = a_2 = a_3 = 0$

vectors are linearly independent

$L(S) = V_3(R)$

So set of vectors for basis of $\underline{R}^3$

let $V_3(R)$ be a finite dimensional vector space

Find the coordinate vector of $\alpha = (3, 1, -4)$

relative to the basis $\beta = \{\alpha_1 \alpha_2 \alpha_3\}$

where $\alpha_1 = (1,1,1)$ $\alpha_2 = (0,1,1)$ $\alpha_3 = (0,0,1)$

$\alpha = c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3$

$(3, 1, -4) = c_1(1,1,1) + c_2(0,1,1) + c_3(0,0,1)$

$c_1 = 3$

$c_1 + c_2 = 1 \implies 3 + c_2 = 1$

$c_2 = 1 - 3 = -2$

$c_1 + c_2 + c_3 = -4$

$3 - 2 + c_3 = -4$

$1 + c_3 = -4$

$c_3 = -5$